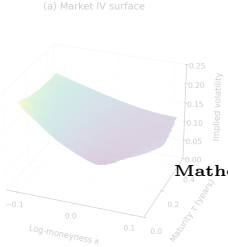
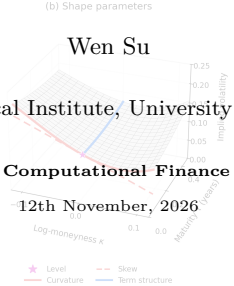
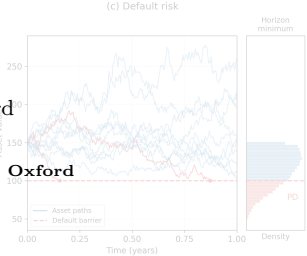
Credit Risk Behind The Smile







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Outline

1. Introduction
2. Structural Model
3. Implied Volatility Expansion
4. Default Probability Identification
5. Estimation Methodology
6. Simulation Study
7. Empirical Study
8. Conclusion

Estimating credit risk when direct information is unavailable

- Since the 2008 global financial crisis, measuring corporate credit risk has become an important task for **investors, dealer banks, and regulators**.
- **Credit default swap (CDS)** spreads and **credit ratings** are widely used to assess credit risk, but they are unavailable for many firms (Faulkender and Petersen (2006, RFS); Subrahmanyam et al. (2014, RFS)).
- Basel Committee on Banking Supervision, 2020:

  Banks must have a methodology for estimating default probabilities for counterparties whose credit is not actively traded.

Existing Methods

- **Structural models.** Derive default probabilities and credit spreads from **asset dynamics, default rules, and debt obligations**.
  - **Merton and first-passage models**: default at maturity or when asset value hits a default boundary (Merton (1974, JF); Black and Cox (1976, JF)).
  - **Stochastic volatility and jumps** in firm asset values (Cremers et al. (2008, RFS); Collin-Dufresne et al. (2012, JF); Du et al. (2019, JF)).
  - **Debt dynamics**: mean-reverting leverage (Collin-Dufresne and Goldstein (2001, JF)) and stochastic debt (Feldhütter and Schaefer (2023, JFE)).
- **Statistical methods.** Predict default from **observable information**.
  - **Financial information**: Z-score (Altman (1968, JF)) and O-score (Ohlson (1980, JAR)).
  - **Financial and market information**: combine firm financials with stock returns and volatility (Campbell et al. (2008, JF); Hilscher and Wilson (2017, MS)).

The equity option market may be useful

	Number	Market	Source
Listed companies	12,134	U.S., 2025	SEC (2026)
Option coverage	8,439 (69.5%)	U.S., 2025	SEC (2026)
S&P-rated companies	$\approx$2,300	Global, 2026	S&P Global (2026)
Single-name CDS	725	Global, 2024 Q2	ISDA (2024)

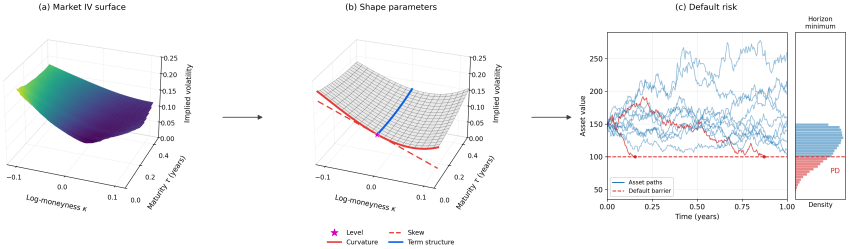
Notes: SEC = U.S. Securities and Exchange Commission; ISDA = International Swaps and Derivatives Association.

- Equity options and CDS provide **risk-neutral information** about firms (Aramonte et al. (2022, MS); Cao et al. (2023, MS)).
- We aim to use equity implied volatility (IV) to predict credit risk.

Challenges in the literature

- Existing researchers study the link between credit and equity options or IVs (Hull et al. (2005, JCR); Cremers et al. (2008, RFS); Carr and Wu (2011, RFS); Elkhodiry et al. (2011, AOR); Berndt et al. (2018, RoF); Culp et al. (2018, AER); Mercadier and Lardy (2019, EJOR); Cao et al. (2023, MS)).
- **Challenges in using equity IV:**
  1. **Which IV points should we use?**
     IV varies across strikes and maturities.
  2. **How do we bridge the maturity gap?**
     Available options are often short-dated. How can short-maturity IV information be used to estimate a five-year CDS spread?
  3. **Why should IV predict credit risk?**
     Why does IV contain information about credit risk?

This paper



- **Two-step estimation**:
  - **Step 1**: Compress the IV surface into shape parameters $\boldsymbol{\Theta}$.
  - **Step 2**: Find a mapping from $\boldsymbol{\Theta}$ to credit risk.
- We develop a two-step non-parametric method for estimating credit risk from the IV surface and provide a theoretical foundation based on a general structural model.

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A review of Merton's model

- Merton (1974, JF): Suppose that $\mathrm{d}A_t = rA_t\mathrm{d}t + \sigma_A A_t \mathrm{d}B_t$. Then

$$\mathrm{PD} = \mathbb{Q}\left(A_T \leq D\right) = 1 - N(d_2),$$

  where $d_2 = -\frac{\log L}{\sigma_A\sqrt{T}} - \frac{1}{2}\sigma_A\sqrt{T}$ with $L = D/A_0$.

- The firm's asset value A_t is unobserved. The model treats equity as a call option on A_t:

$$E_t = A_t N(d_1) - e^{-r(T-t)} D N(d_2).$$

  By Itô's lemma,

$$\sigma_E = \frac{\sigma_A N(d_1)}{N(d_1) - L N(d_2)}.$$

- This suggests that the asset-value distribution may be inferred from the equity-value distribution. In more complex models, however, this relationship may not have an explicit form.

Asset dynamics

- Our framework starts from a general stochastic volatility model. And:

  All quantities must be derived from the same underlying asset dynamics.

- Let A_t denote the firm's **asset value** and $V_t > 0$ its **instantaneous return variance**. Under a risk-neutral measure $\mathbb{Q}$,

$$\begin{aligned} \frac{\mathrm{d}A_t}{A_t} &= (r_t - q_t)\,\mathrm{d}t + \sqrt{V_t}\,\mathrm{d}W_t, \\ \mathrm{d}V_t &= \mu(V_t)\,\mathrm{d}t + \xi(V_t)\left\{\rho\,\mathrm{d}W_t + \sqrt{1-\rho^2}\,\mathrm{d}W_t^{\perp}\right\}. \end{aligned} \tag{1}$$

- r_t and q_t are deterministic interest and asset payout rates.

- The functions μ and ξ govern **variance drift and diffusion**, with $\xi(v) > 0$. Their functional forms are left unspecified, subject to conditions ensuring a positive, nonexplosive variance process.

Default and the transformed state

- Let $D_u > 0$ be a deterministic, continuously differentiable **default threshold**. Default occurs when asset value first reaches this threshold.
- Starting from a surviving firm at time t, define the **log asset-to-threshold ratio** and default time:

$$X_u = \log\left(\frac{A_u}{D_u}\right), \qquad \tau_D = \inf\{u \geq t : X_u \leq 0\}. \tag{2}$$

  By Itô's formula,

$$\mathrm{d}X_u = \left(r_u - q_u - \frac{\dot{D}_u}{D_u} - \frac{V_u}{2}\right)\mathrm{d}u + \sqrt{V_u}\,\mathrm{d}W_u. \tag{3}$$

- Equity values, equity IV, and credit risk must all be derived from these underlying asset dynamics.

Equity dynamics

- Equity is an **option-like claim** on the firm's assets. Before default, its value is represented by a smooth pricing map $S_t = s(t, X_t, V_t)$. Equity value is zero after default.
- Assume that equity has the same proportional payout rate q_t as assets. The pricing map satisfies

$$(\partial_t + \mathcal{L}_t)s = (r_t - q_t)s, \tag{4}$$

where $\mathcal{L}_t$ is the generator of (X_t, V_t):

$$\mathcal{L}_t f = \left(r_t - q_t - \frac{\dot{D}_t}{D_t} - \frac{v}{2}\right) f_x + \mu(v) f_v + \frac{v}{2} f_{xx} + \rho\sqrt{v}\,\xi(v) f_{xv} + \frac{\xi(v)^2}{2} f_{vv}. \tag{5}$$

- A typical choice is a **Black–Cox-type equity claim** (Black and Cox (1976, JF)):

$$s(t, x, v) = \mathbb{E}^{\mathbb{Q}}_{t,x,v}\left[e^{-\int_t^{T_E}(r_u - q_u)\,\mathrm{d}u}(A_{T_E} - D_{T_E})^+ \mathbf{1}_{\{\tau_D > T_E\}}\right], \tag{6}$$

where $T_E > t$ is the equity valuation horizon.

Equity dynamics

- At the current state $(X_t, V_t) = (x, v)$ and before default, Itô's formula gives

$$\frac{\mathrm{d}S_t}{S_t} = (r_t - q_t)\,\mathrm{d}t + \frac{\sqrt{v}\,s_x + \rho\xi(v)s_v}{s}\,\mathrm{d}W_t + \frac{\sqrt{1-\rho^2}\,\xi(v)s_v}{s}\,\mathrm{d}W_t^{\perp}. \tag{7}$$

  Here s and its derivatives are evaluated at (t, x, v).

- The **instantaneous equity variance** is therefore

$$\begin{aligned}\sigma_S^2(t,x,v) &= \frac{vs_x^2 + 2\rho\sqrt{v}\xi(v)s_x s_v + \xi(v)^2 s_v^2}{s^2} \\ &= v\left(\frac{s_x}{s}\right)^2 + 2\rho\sqrt{v}\xi(v)\frac{s_x}{s}\frac{s_v}{s} + \xi(v)^2\left(\frac{s_v}{s}\right)^2.\end{aligned} \tag{8}$$

  The ratio s_x/s is the elasticity of equity value with respect to asset value, while s_v/s is its proportional sensitivity to variance.

- We have: Asset (✓), Equity (✓), IV (✗), Credit (✗).

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Equity options under the structural model

- Consider a **European call on equity** with strike $K > 0$ and maturity $T > t$. Its value at the current state $(X_t, V_t) = (x, v)$ is

$$P(t, x, v; T, K) = \mathbb{E}^{\mathbb{Q}}_{t,x,v}\left[e^{-\int_t^T r_u \,\mathrm{d}u}(S_T - K)^+\right], \tag{9}$$

  where $S_T = 0$ if default occurs before T.

- Define **time to maturity** and **forward log-moneyness** by

$$\tau = T - t, \qquad \kappa = \log\left(\frac{K}{S_t \exp\{\int_t^T (r_u - q_u)\,\mathrm{d}u\}}\right). \tag{10}$$

  The point $\kappa = 0$ corresponds to an at-the-money forward option.

- The **equity implied volatility** is the value satisfying

$$P(t, x, v; T, K) = P^{\mathrm{BS}}\left(t, S_t; T, K, \sigma_{\mathrm{IV}}(t, x, v; \kappa, \tau)\right), \tag{11}$$

  using the same interest and payout rates in the Black–Scholes formula.

A local IV expansion

- Given the equity dynamics, IV can be expanded as a power series in (κ, τ) (Lorig et al. (2017, MF); Pagliarani and Pascucci (2017, FS); Aït-Sahalia et al. (2021, RFS)):

$$\tau \downarrow 0, \qquad |\kappa| \leq \lambda\sqrt{\tau}, \qquad \lambda > 0. \tag{12}$$

Order	Terms	Size
0	1	$O(1)$
1	κ	$O(\tau^{1/2})$
2	$\kappa^2,\ \tau$	$O(\tau)$
3	$\kappa^3,\ \kappa\tau$	$O(\tau^{3/2})$

- We apply these techniques to the equity dynamics implied by our structural model.

Theorem 1: local IV expansion

Assumption

The functions $r(t), q(t), D(t), \mu(v), \xi(v)$ and $s(t,x,v)$ are C^∞ on a neighbourhood of a compact subset $\mathcal{K} \subset [0, T_E) \times (0,\infty)^2$.

Theorem (Local IV expansion)

For a fixed integer $N \geq 0$ and $\lambda > 0$, there exist unique coefficients $\theta_{jm}(t,x,v)$ such that

$$\sigma_{\mathrm{IV}}(t,x,v;\kappa,\tau) = \sum_{\substack{j,m\geq 0 \\ j+2m\leq N}} \theta_{jm}(t,x,v)\kappa^j\tau^m + R_N(t,x,v;\kappa,\tau), \tag{13}$$

where, for sufficiently small τ, $\sup_{\substack{(t,x,v)\in\mathcal{K} \\ |\kappa|\leq\lambda\sqrt{\tau}}} |R_N(t,x,v;\kappa,\tau)| \leq C_{N,\mathcal{K},\lambda}\tau^{(N+1)/2}$.

$N = 2$: Four IV shape coefficients

- Taking $N = 2$ in Theorem 1 gives

$$\sigma_{\mathrm{IV}}(\kappa, \tau) = \theta_{00} + \theta_{10}\kappa + \theta_{20}\kappa^2 + \theta_{01}\tau + O(\tau^{3/2}). \tag{14}$$

  The approximation holds uniformly for $|\kappa| \leq \lambda\sqrt{\tau}$.

- The four coefficients describe the **local shape of the IV surface**:

Coefficient	IV feature	Interpretation
θ_{00}	**Level**	Short-maturity ATM IV
θ_{10}	**Skew**	Strike slope
θ_{20}	**Curvature**	Smile curvature
θ_{01}	**Maturity dependence**	Term-structure slope

- Define the **shape parameter vector**:

$$\boldsymbol{\Theta}_2 = (\theta_{00}, \theta_{10}, \theta_{20}, \theta_{01})^\top. \tag{15}$$

$N = 2$: Structural interpretation

- Define the **instantaneous covariance operator**

$$\Gamma(f, g) = v f_x g_x + \rho\sqrt{v}\,\xi(v)(f_x g_v + f_v g_x) + \xi(v)^2 f_v g_v, \tag{16}$$

  and let $\mathcal{D}f = \Gamma(\log s, f)$.

- The four IV coefficients admit the following expressions:

$$\begin{aligned}
\theta_{00} &= \sigma_S, \qquad \theta_{10} = \frac{\mathcal{D}\sigma_S}{2\sigma_S^2}, \\
\theta_{20} &= \frac{2\sigma_S\mathcal{D}^2\sigma_S - 7(\mathcal{D}\sigma_S)^2 + 2\sigma_S^2\Gamma(\sigma_S, \sigma_S)}{12\sigma_S^5}, \\
\theta_{01} &= \frac{1}{2}(\partial_t + \mathcal{L}_t)\sigma_S + \frac{1}{4}\mathcal{D}\sigma_S - \frac{\mathcal{D}^2\sigma_S}{6\sigma_S^2} + \frac{5(\mathcal{D}\sigma_S)^2}{24\sigma_S^3} + \frac{\Gamma(\sigma_S, \sigma_S)}{12\sigma_S}.
\end{aligned} \tag{17}$$

- Key takeaways:
  Asset dynamics determine the IV coefficients: $(x, v, \mu(\cdot), \xi(\cdot), \rho) \mapsto \boldsymbol{\Theta}_N$.
  Specifically, $(x, v, \mu(v), \mu'(v), \cdots, \mu^{(N)}(v), \xi(v), \xi'(v), \cdots, \xi^{(N)}(v), \rho) \mapsto \boldsymbol{\Theta}_N$.

- We have: Asset (✓), Equity (✓), IV (✓), Credit (✗).

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From IV shape to default probability

- For a structural model M, the **risk-neutral default probability** over a fixed horizon $H > 0$ is
$$\mathrm{PD}_H(M) = \mathbb{Q}^M_{t,x_M,v_M}(\tau_D \leq t + H). \tag{18}$$
- We have established the mapping $(x_M, v_M, \mu_M(\cdot), \xi_M(\cdot), \rho_M) \mapsto \boldsymbol{\Theta}_N(M)$.
- **Identification question:**

  Do similar IV shape coefficients imply similar default probabilities?

Conditions for default probability identification

Assumption (Structural specification)

1. *Equity is valued by the Black–Cox-type claim introduced above. The default threshold satisfies* $D_t > 0$ *and* $\dot{D}_u = (r_u - q_u)D_u$ *for* $u \geq t$.
2. $\rho \in (-1, 0)$.

Write $Y_u = \log V_u$ and $y = \log v$, and define

$$\beta(y) = e^{-y}\xi(e^y), \quad \alpha(y) = e^{-y}\mu(e^y) - \tfrac{1}{2}\beta(y)^2, \quad h(y) = \log \beta(y). \tag{19}$$

Assumption (Analytic class)

1. *The initial state* (x, y) *belongs to a common compact set* $\mathcal{K}_1 \subset (0, \infty) \times \mathbb{R}$.
2. $\alpha(\cdot)$ *and* $h(\cdot)$ *are analytic and bounded.*
3. *The variance diffusion coefficient satisfies*

$$h'(y) \leq \frac{1}{2} \quad \forall y \in \mathbb{R} \quad \Longleftrightarrow \quad v \mapsto \frac{\xi(v)}{v^{3/2}} \text{ is non-increasing on } (0, \infty). \tag{20}$$

Theorem 2: PD Identification

- For $N \geq 2$, let $\mathbf{\Theta}_N(M) = (\theta_{jm}(M) : j + 2m \leq N)$. Define the **coefficient discrepancy** and **effective expansion order**:

$$\varepsilon_N = \|\mathbf{\Theta}_N(M_1) - \mathbf{\Theta}_N(M_2)\|_2, \quad N_{\text{eff}} = \min\{N, \log_+(1/\varepsilon_N)\}, \tag{21}$$

  where $\log_+(u) = \max\{\log u, 0\}$ and $N_{\text{eff}} = N$ when $\varepsilon_N = 0$.

Theorem (Default probability identification)

Let M_1, M_2 be admissible models satisfying the preceding conditions.
(a) For every fixed $H > 0$, if $\mathbf{\Theta}_N(M_1) = \mathbf{\Theta}_N(M_2)$ for all N, then $\mathrm{PD}_H(M_1) = \mathrm{PD}_H(M_2)$.
(b) Moreover, there exist constants $C, c > 0$, independent of N, such that

$$|\mathrm{PD}_H(M_1) - \mathrm{PD}_H(M_2)| \leq C \exp\left\{-c[\log\log(N_{\text{eff}} + 3)]^2\right\}, \qquad N \geq 2. \tag{22}$$

- Under the analyticity conditions and the time-consistent specification, local derivative information can be sufficient to characterise $(\mu(v), \xi(v))$.

- We have: Asset (✓), Equity (✓), IV (✓), Credit (✓).

Corollary: Smooth IV-to-PD mapping

- Theorem 2 implies that **finite-order IV coefficients** can approximate fixed-horizon default probabilities through a smooth mapping.
- **GBM illustration**: With constant asset variance, the mapping is explicit:

$$\begin{aligned} \mathrm{PD}_H = \Phi\left(\frac{\log(-2\theta_{10}/\theta_{00})}{(\theta_{00}+2\theta_{10})\sqrt{H}} + \frac{(\theta_{00}+2\theta_{10})\sqrt{H}}{2}\right) \\ + \frac{\theta_{00}}{-2\theta_{10}}\,\Phi\left(\frac{\log(-2\theta_{10}/\theta_{00})}{(\theta_{00}+2\theta_{10})\sqrt{H}} - \frac{(\theta_{00}+2\theta_{10})\sqrt{H}}{2}\right), \end{aligned} \tag{23}$$

- With **stochastic asset variance**, the mapping need not admit a closed-form expression.

Corollary: Smooth IV-to-PD mapping

Corollary (Smooth approximation)

For each $N \geq 2$, there exists $g_N \in C^\infty(\mathbb{R}^{d_N})$, where $d_N = \dim(\boldsymbol{\Theta}_N)$, such that

$$\sup_{M \in \mathcal{M}} |\mathrm{PD}_H(M) - g_N(\boldsymbol{\Theta}_N(M))| \leq C \exp\left\{-c[\log\log(N+3)]^2\right\}. \tag{24}$$

- The smooth mapping can be expanded as a power series:

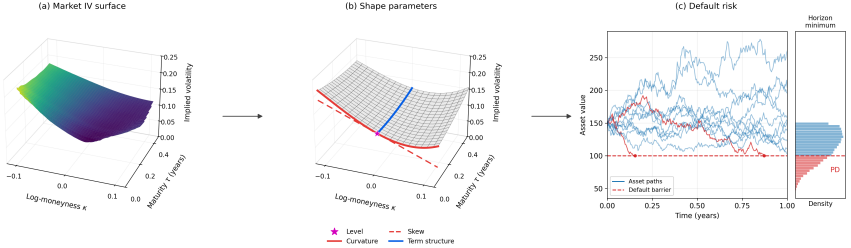
$$g_N(\boldsymbol{\Theta}_N) \approx \alpha + \boldsymbol{\beta}^\top \boldsymbol{\Theta}_N + \boldsymbol{\Theta}_N^\top B \boldsymbol{\Theta}_N + \sum_{a \leq b \leq c} \gamma_{abc} \Theta_{N,a} \Theta_{N,b} \Theta_{N,c} + \cdots. \tag{25}$$

- In general, the credit spread is also a smooth function of PD and therefore inherits these properties.
- **This motivates our two-step estimation**: first estimate $\boldsymbol{\Theta}_N$, then use these coefficients to estimate credit risk.

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Two-step estimation



- **Two-step estimation:**
  1. **IV estimation**: estimate $\widehat{\boldsymbol{\Theta}}_N$ from equity option quotes across strikes and maturities.
  2. **Credit estimation**: learn the relationships between $\widehat{\boldsymbol{\Theta}}_N$ and observed CDS spreads or credit ratings using firms with available credit information.
- Once the relationship has been estimated, it can be applied to a firm with no available CDS spread or credit rating, using its option data.

Step 1: Estimating IV shape coefficients

- For each firm and date, we observe n equity option quotes $\{(\kappa_i, \tau_i, \sigma^{\text{obs}}_{\text{IV},i})\}_{i=1}^{n}$.
- Recall the **local polynomial representation**:

$$\sigma^{\text{obs}}_{\text{IV},i} = \sum_{j+2m\leq N} \theta_{jm} \kappa_i^j \tau_i^m + R_N(\kappa_i, \tau_i) + \epsilon_i, \tag{26}$$

  where ϵ_i captures observation errors.
- Following Aït-Sahalia et al. (2021, RFS), we estimate $\boldsymbol{\Theta}_N$ by **weighted least squares**:

$$\widehat{\boldsymbol{\Theta}}_N = \arg\min_{\boldsymbol{\vartheta}} \sum_{i=1}^{n} \omega_i \left(\sigma^{\text{obs}}_{\text{IV},i} - \sum_{j+2m\leq N} \vartheta_{jm} \kappa_i^j \tau_i^m \right)^2. \tag{27}$$

- Weights are inversely proportional to the number of quotes at each maturity, so that **every maturity receives equal total weight**.

Step 2: Predicting CDS spreads

- For a fixed CDS maturity H, we estimate the relationship

$$\log \mathrm{CDS}_{H,i}^{\mathrm{obs}} = f(\boldsymbol{\Theta}_{N,i}) + u_i, \tag{28}$$

  using firm–date observations with available CDS spreads.

- We approximate f by a **polynomial of total degree** P:

$$f_P(\boldsymbol{\Theta}_N) = \alpha + \boldsymbol{\beta}^\top \boldsymbol{\phi}_P(\boldsymbol{\Theta}_N), \quad \boldsymbol{\phi}_P(\boldsymbol{\Theta}_N) = \left(\prod_{a=1}^{d_N} \Theta_{N,a}^{\nu_a} \right)_{1 \le |\boldsymbol{\nu}| \le P}. \tag{29}$$

  Here $d_N = \dim(\boldsymbol{\Theta}_N)$, $\boldsymbol{\nu} \in \mathbb{N}_0^{d_N}$, and $|\boldsymbol{\nu}| = \sum_{a=1}^{d_N} \nu_a$. The feature vector contains all powers and interactions up to degree P, excluding the constant.

- Polynomial expansion can give a lot of terms, we estimate the coefficients by **LASSO**:

$$\min_{\alpha, \boldsymbol{\beta}} \quad \sum_i \left[\log \mathrm{CDS}_{H,i}^{\mathrm{obs}} - \alpha - \boldsymbol{\beta}^\top \boldsymbol{\phi}_P(\widehat{\boldsymbol{\Theta}}_{N,i}) \right]^2 + \lambda_{\mathrm{c}} \|\boldsymbol{\beta}\|_1. \tag{30}$$

  The intercept is unpenalised.

- The predicted log-CDS spread is $\widehat{f}_P(\widehat{\boldsymbol{\Theta}}_N) = \widehat{\alpha} + \widehat{\boldsymbol{\beta}}^\top \boldsymbol{\phi}_P(\widehat{\boldsymbol{\Theta}}_N)$.

Step 2: Predicting credit ratings

- Let $R_i \in \{1, \dots, K\}$ denote the observed credit-rating category of firm–date observation i.
- We estimate **rating probabilities** using multinomial logistic regression:

$$p_{ik} = \mathbb{P}(R_i = k \mid \boldsymbol{\Theta}_{N,i}) = \frac{\exp\{f_{k,P}(\boldsymbol{\Theta}_{N,i})\}}{\sum_{\ell=1}^{K} \exp\{f_{\ell,P}(\boldsymbol{\Theta}_{N,i})\}}. \tag{31}$$

- Each rating category has a **polynomial score of total degree** P, using the same feature vector:

$$f_{k,P}(\boldsymbol{\Theta}_N) = \alpha_k + \boldsymbol{\beta}_k^{\top} \boldsymbol{\phi}_P(\boldsymbol{\Theta}_N), \qquad k = 1, \dots, K-1. \tag{32}$$

  We set $f_{K,P} = 0$ for the reference category.
- We estimate the coefficients by ℓ_1**-regularised maximum likelihood**:

$$\min_{\{\alpha_k, \boldsymbol{\beta}_k\}_{k=1}^{K-1}} \left\{ -\sum_i \log p_{i,R_i} + \lambda_{\mathrm{r}} \sum_{k=1}^{K-1} \|\boldsymbol{\beta}_k\|_1 \right\}, \tag{33}$$

  replacing $\boldsymbol{\Theta}_{N,i}$ by $\widehat{\boldsymbol{\Theta}}_{N,i}$. The intercepts are unpenalised.
- The predicted rating is $\widehat{R}_i = \arg\max_k \widehat{p}_{ik}$.

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Simulation design

We generate **500 firms** from four asset models, each selected independently with probability 1/4.

Model	Asset and volatility dynamics	Parameter distributions
GBM	$\mathrm{d}A_t/A_t = \sigma_A \,\mathrm{d}W_t$	$\sigma_A \sim U(0.15, 0.35)$
CEV	$\mathrm{d}A_t = \alpha A_t^{\beta}\,\mathrm{d}W_t$	$\alpha \sim U(0.225, 0.375)$, $\beta \sim U(0.60, 0.80)$
SABR	$\mathrm{d}A_t = \alpha_t A_t^{\beta}\,\mathrm{d}W_t,\ \mathrm{d}\alpha_t = \nu\alpha_t\,\mathrm{d}Z_t$	$\alpha_0 \sim U(0.225, 0.375)$, $\beta \sim U(0.60, 0.80)$, $\nu \sim U(0.20, 0.30)$
Heston	$\mathrm{d}A_t/A_t = \sqrt{V_t}\,\mathrm{d}W_t,\ \mathrm{d}V_t = \kappa(\bar{v} - V_t)\,\mathrm{d}t + \xi\sqrt{V_t}\,\mathrm{d}Z_t$	$v_0 \sim U(0.0225, 0.1225)$, $\kappa \sim U(1, 2)$, $\bar{v} \sim U(0.05, 0.075)$, $\xi \sim U(0.20, 0.30)$

- **Common parameters:** $x_0 = \log(A_0/D) \sim U(0.35, 1.00)$; $\rho \sim U(-0.7, -0.3)$ for SABR and Heston.
- All models use **first-passage default** at $D = 1$. We set $r = q = 0$ and recovery to 40%.
- **Task**: even without knowing the asset dynamics form, can our procedure accurately estimate credit spread from IV surface?

Simulation: IV and CDS generation

- **Equity options:** For each firm, we price European equity options at six maturities,

$$\tau \in \{15, 30, 45, 90, 120, 180\}/365, \qquad \kappa = z\sqrt{\tau}, \quad z = -0.40, -0.36, \ldots, 0.40. \tag{34}$$

  This gives **126 IV quotes per firm** (six maturities and 21 strikes each).

- **Five-year CDS:** The same asset model determines default probabilities and spreads:

$$\mathrm{CDS}_{5,i} = 10^4(1 - \mathrm{Rec})\frac{\mathrm{PD}_{5,i}}{\int_0^5 (1 - \mathrm{PD}_{u,i})\,\mathrm{d}u}. \tag{35}$$

- **Evaluation:**
  - In-sample and out-of-sample (five-fold cross-validation).
  - RMSE: $\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\widehat{\mathrm{CDS}_{5,i}} - \mathrm{CDS}_{5,i}\right)^2}$; MAE: $\frac{1}{n}\sum_{i=1}^{n}\left|\widehat{\mathrm{CDS}_{5,i}} - \mathrm{CDS}_{5,i}\right|$.

Simulation results

	In-sample			Out-of-sample		
Model	RMSE	MAE	R^2	RMSE	MAE	R^2
Raw IV LASSO	124.01	86.86	0.859	129.98	90.84	0.832
$N = 2$, Linear	238.00	121.59	0.372	242.24	123.36	0.329
$N = 2$, Quadratic	78.22	54.15	0.934	87.88	58.06	0.920
$N = 2$, Cubic	46.38	30.83	0.979	57.86	36.32	0.929
$N = 3$, Linear	212.23	114.17	0.520	227.37	118.60	0.457
$N = 3$, Quadratic	65.79	45.60	0.957	79.38	52.28	0.931
$N = 3$, Cubic	34.21	24.06	0.989	55.17	31.60	0.956

- **LASSO selection:** The best-performing $N = 3$ Cubic model retains 38 out of 83 polynomial terms.
- All six IV shape coefficients enter the fitted model, either directly or through nonlinear interactions.

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6. Simulation Study
7. Empirical Study
8. Conclusion

Data and prediction task

- **Data**:
  - We use daily data from the J.P. Morgan dataset, covering **January 2018 to December 2025** for **397 firms**.
  - Equity IV surfaces, CDS spreads (5 year), and S&P credit ratings are matched by firm and date.
- **IV shape parameters**:
  - For each **firm–date**, we estimate the IV shape parameters separately using the **second-order IV expansion**:

$$\widehat{\sigma}_{\mathrm{IV}}(\kappa,\tau) = \widehat{\theta}_{00} + \widehat{\theta}_{10}\kappa + \widehat{\theta}_{20}\kappa^2 + \widehat{\theta}_{01}\tau. \tag{36}$$

  - This step 1 gets overall 85% R^2.
- **Task**:
  - We use **date-wise leave-one-firm-out prediction**: train on the remaining firms at each date and use the held-out firm's credit outcomes only for evaluation.
  - Predict 5-year CDS spreads and credit ratings.

Empirical Results

Table 1: Leave-one-out Empirical Results

	CDS spread			Credit rating		
Model	R^2	RMSE	MAE	Accuracy	MAE	Macro-F_1
Panel A: Credit Information						
Rating / CDS	0.4779	56.38	29.36	0.5335	0.5091	0.4419
Panel B: Structural Benchmarks						
Merton (1974, JF)	0.3953	60.67	36.08	0.4482	0.6067	0.1671
Du et al. (2019, JF)	0.4097	59.95	33.27	0.4634	0.5854	0.2102
Chen et al. (2026, JF)	0.2067	69.49	28.67	0.4543	0.5945	0.2282
Panel C: IV Surface						
Raw IV LASSO	-0.6661	100.71	32.34	–	–	–
IV Shape	0.5573	51.91	26.90	0.5061	0.5244	0.3445

Errors across maturities

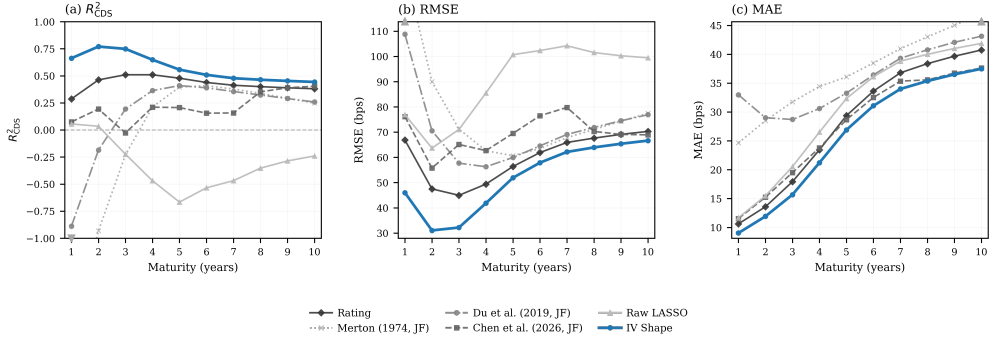


Figure 1: Errors across maturities

Empirical interpretation of IV coefficients

- Consider the $N = 2$ Cubic specification. We calculate the **average marginal slope** of predicted log CDS with respect to each standardised IV coefficient.

IV shape	Average slope	Positive slopes (%)
Level θ_{00}	$+0.722$	99.4
Skew θ_{10}	-0.025	64.0
Curvature θ_{20}	-0.078	2.7
Maturity θ_{01}	$+0.293$	99.7

- **Level:** Higher equity IV is strongly associated with higher CDS spreads.
- **Skew:** The average slope is close to zero, with mixed signs across observations.
- **Curvature:** Higher IV curvature is associated with lower CDS spreads in most observations.
- **Maturity:** A positive relationship with CDS spreads is observed consistently.

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Conclusion

- Credit risk must often be estimated when CDS spreads and credit ratings are unavailable.
- We propose a two-step procedure that summarises equity IV surfaces by shape coefficients and maps them to credit risk.
- Theoretically, under the stated conditions, the complete coefficient sequence identifies risk-neutral default probabilities, with a finite-order stability bound.
- Empirically, IV shape predicts CDS spreads and credit ratings more accurately than the structural benchmarks, without using the target firm's credit information.
- Further work will develop statistical inference for the two-step procedure and evaluate its performance for firms without observed credit information.

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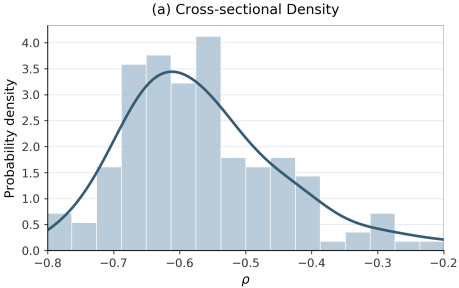
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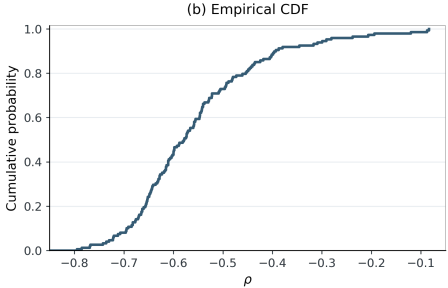
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Cross-sectional correlation estimates

- We calibrate a **Heston asset model separately for each firm**, jointly matching the six estimated IV coefficients $\widehat{\boldsymbol{\Theta}}_3$ and the observed five-year CDS spread. All parameters, including ρ, are estimated independently across firms.





- The estimates cluster around -0.6, with a **mean of** -0.556, a median of -0.579, and a standard deviation of 0.135.